DC Motor.

The DC moto r is not so used nowadays as it was in the past. For most application, it has been replaced by the solid-state rectifiers. Figure 1 shows an elementary machine equipped with a field winding wound on the stator poles, a rotor coil and a commutator:

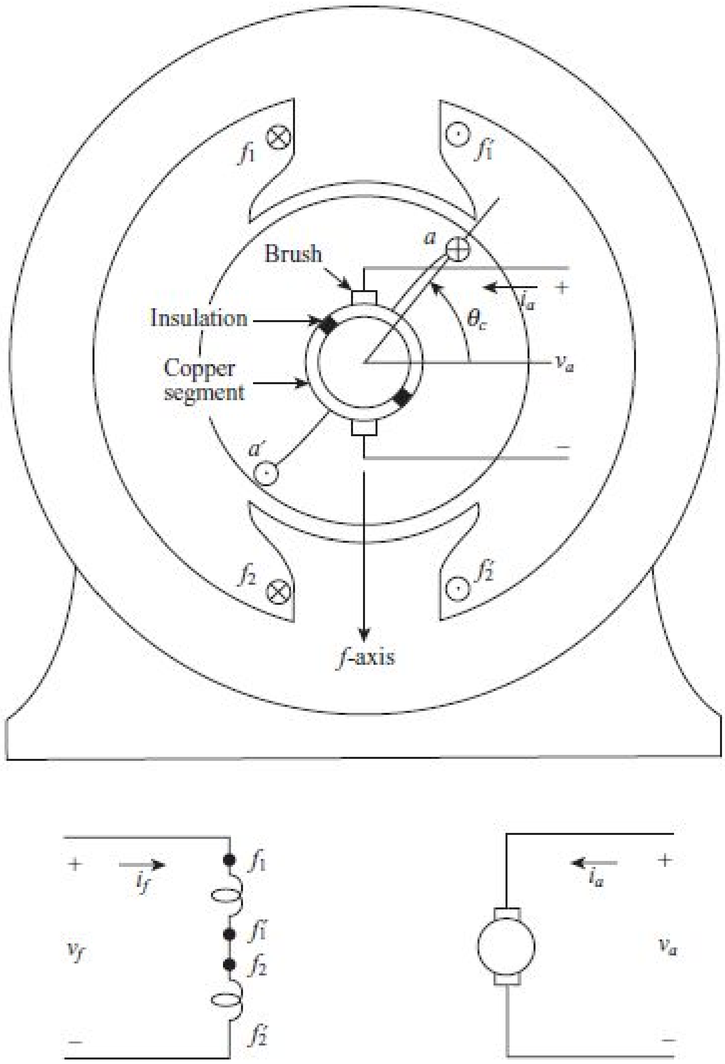

Figure 1. Elementary two-pole DC Machine [1]

The commutator is made up of two semi-circular copper segments mounted on the shaft at the end of the rotor.These segments are insulated from one another as well as from the iron of the rotor. Each terminal of the rotor coil is connected to a copper segment. Stationary carbon brushes ride upon the copper segments whereby the rotor coil is connected to a stationary circuit.

The voltage equations for the field winding and rotor coil are:

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$v_{a-a'} = r_a i_{a-a'} + \frac{d\lambda_{a-a'}}{dt}$$

Equations 1

The flux linkage is expressed as:

$$\lambda_f = L_{ff} i_f + L_{fa} i_{a-a'}$$

$$\lambda_{a-a'} = L_{af} i_f + L_{aa} i_{a-a'}$$

Equations 2

R_f and R_a are the resistances of the field winding and the armature coil. The armature is the term used to refer to the rotor, so both mean the same. The mutual inductance between the field winding and the armature coil is expressed in term of a sinusoidal function of θ_r as:

$$L_{af} = L_{fa} = -L\cos\theta_r$$

Equations 3

where L is a constant. As the rotor revolves, the function of the commutator is to switch the stationary terminals from one terminal of the rotor coil t the other. This commutation occurs at θ_r =0, Π, 2Π, ...At the instant of the switch, the brushe is in contact with both copper segments, so the rotor coil is short-circuited.

The way form of the voltage induced in the open-circuit armature coil during constant-speed operation with a constant field winding current may be determined by setting $I_{a-a'}$ =0 and I_f =constant. Using the expression from equations 1, 2 and 3, we obtain:

$$v_{a-a'} = \omega_r L I_f \sin\theta_r$$

Equations 4

Note that $V_{a-a'}$ =0 at θ_r =0, Π, 2Π because at this stage is happening the commutation. The next Figure illustrates the commutation:

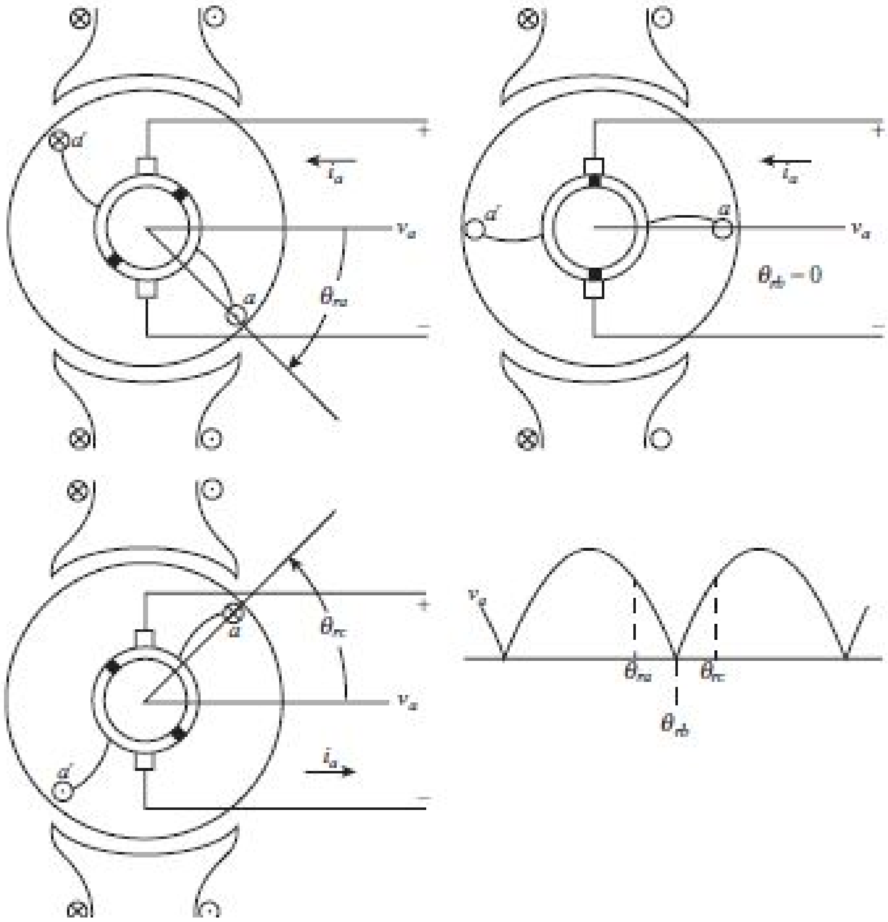

Figure 2. Commutation of the Elementary DC Machine [1]

Note now that the form of V_a makes this configuration an impracticable machine. It could not work effectively as a motor supplied from a voltage source due to the short-circuiting of the armature coil at each commutation.

A more useful machine with 4 pairs of parallel windings is shown in Figure 3, where the rotor is equipped with four **a** windings and with four **A** windings, yielding rectified coil voltages.

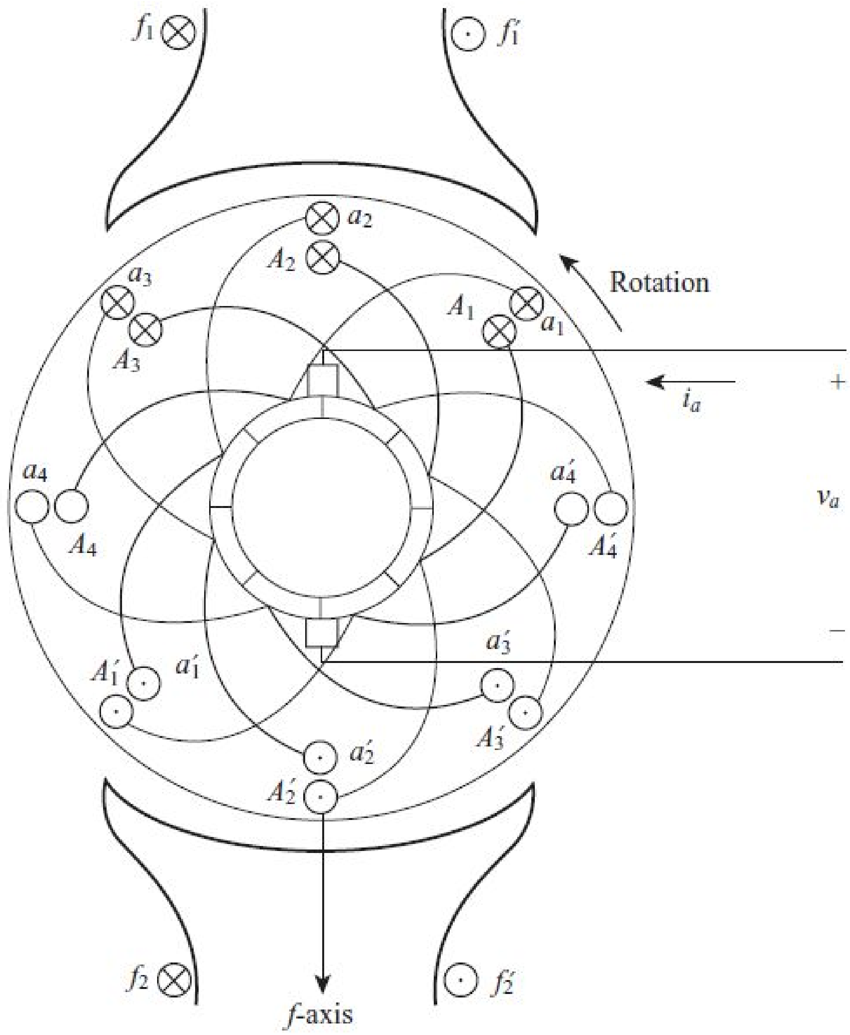

Figure 3. A DC Machine with parallel windings [1]

Now we have that the form of V_a looks like this:

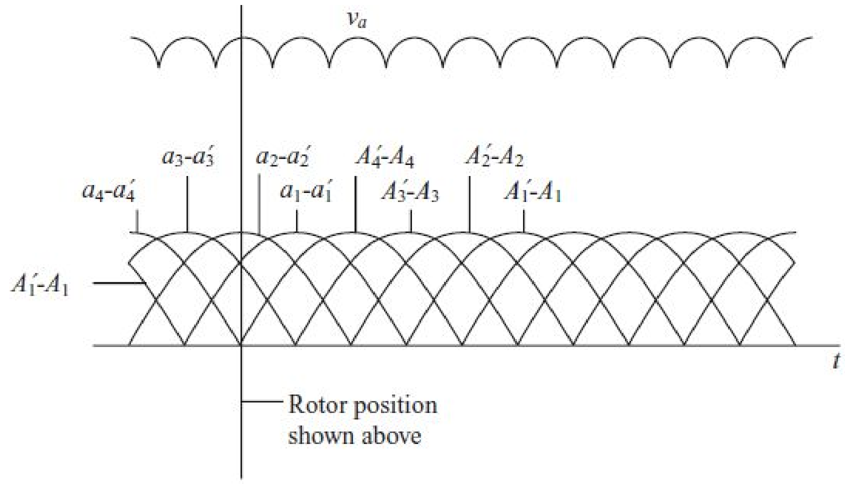

Figure 4. Rectified voltage for a DC Machine with parallel windings [1]

Usually, the number of the rotor coils is more than four reducing by this way the harmonic content of the open-circuit armature voltage V_a . In this case, the rotor coil may be approximated as a uniformly distributed winding. So, the rotor winding is considered as current sheets that are fixed in space due to the action of the commutator and which establish a magnetic axis positioned orthogonal to the magnetic axis of the field winding. This configuration looks as follow:

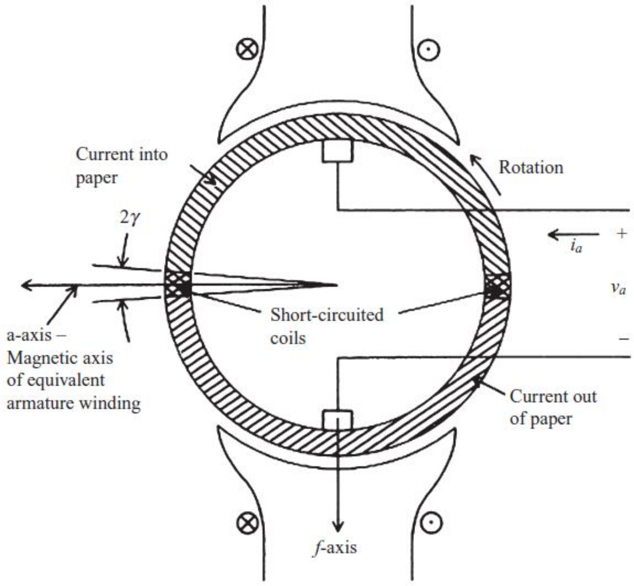

Figure 5. Idealized DC Machine with uniformly distributed rotor winding [1]

Another look for the DC Machine is presented in Figure 6:

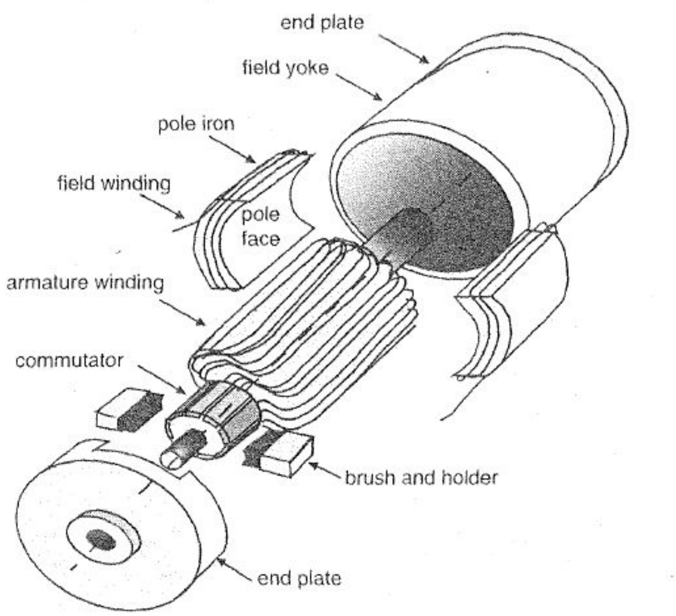

Figure 6. Basic parts of the DC Machine [2]

We can now approximate the equivalent circuit for the idealized DC Machine as:

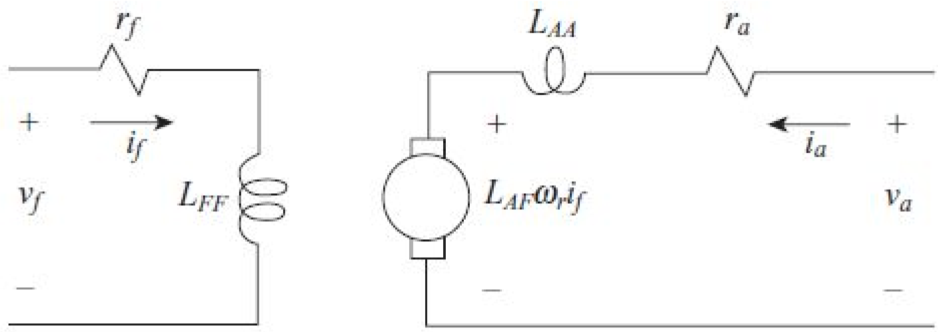

Figure 7. Equivalent circuit for an Idealized DC Machine [1]

From here, we can derive the field and armature voltages which in matrix form look like this:

$$\begin{bmatrix} v_f \\ v_a \end{bmatrix} = \begin{bmatrix} r_f + pL_{FF} & 0 \\ \omega_r L_{AF} & r_a + pL_{AA} \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix}$$

Equations 5

L_{FF} and L_{AA} are self-inductances of the field and armature windings respectively; p is a notation for d/dt; W_r is the rotor speed and L_{AF} is the mutual inductance between the field and the armature. The product $W_r\ L_{AF}\ i_f$ is called back emf (electromotriz force) V_{em}. In this last equation $L_{AF}\ i_f$ is frequently substituted by a constant called K_v:

$$k_v = L_{AF} i_f$$

Equations 6

This substitution is far convenient since even in the case of a permanent-magnet dc machine which has not field circuit, the constant field flux produced by the permanent magnet is analogous to a dc machine with a constant K_v.

We obtain through this important expression for describing the dynamic of DC Motor:

$$V_{em} = K_v\ W_r$$

Equations 7

The above equation dictates that the voltage across the idealized power transducer is proportional to the angular velocity.

For a DC Machine with a field winding, the electromagnetic torque can be expressed as:

$$T_e = L_{AF} i_f i_a$$

Equations 8

Here again we can substitute $L_{AF}\ i_f$ by the constant K_v . So,

$$T_e = K_v\ i_a$$

Equations 9

The electromagnetic torque T_e and the rotor speed are related by:

$$T_e = J\frac{d\omega_r}{dt} + B_m\omega_r + T_L$$

Equations 10

J is the moment of inertia of the rotor and T_L is the load torque, positive for the shaft of the rotor. T_e acts to turn the rotor in the direction of increasing θ_r . The constant B_m is a damping coefficient associated with the mechanical rotational system of the machine.

DC Motors in Control System

The variables and parameters that matter in most of the control system designs are resumed in the following table:

▲ $i_a(t)$ = corriente de armadura	▲ L_a = inductancia de la armadura
▲ R_a = resistencia de armadura	▲ $e_a(t)$ = voltaje aplicado
▲ $e_b(t)$ = fuerza contraelectromotriz	▲ K_b = constante de la fuerza contraelectromotriz
▲ $T_L(t)$ = par de carga	▲ ϕ = flujo magnético en el entre hierro
▲ $T_m(t)$ = par del motor	▲ $\omega_m(t)$ = velocidad angular del rotor
▲ $\theta_m(t)$ = desplazamiento del rotor	▲ J_m = inercia del rotor
▲ K_i = constante del par	▲ B_m = coeficiente de fricción viscosa

Figure 8. Variables and Parameters for a DC Machine [3]

The mode commonly used to represent dc motors in control system literature is as follow:

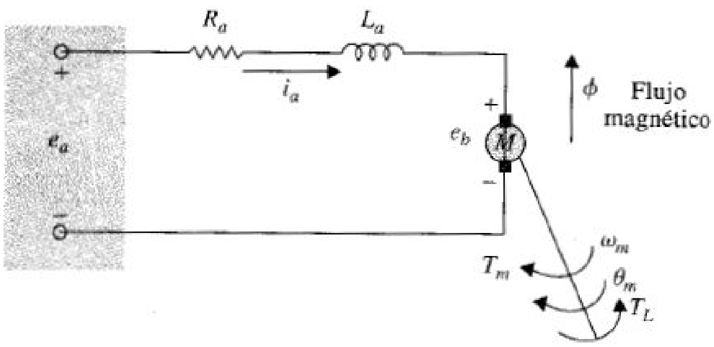

Figure 9. Model for a DC Machine [3]

A variant is presented in Figure 10:

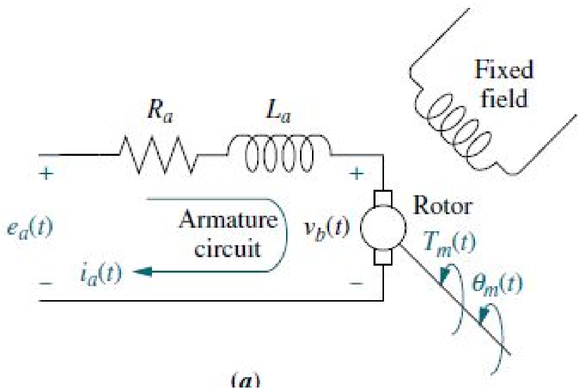

Figure 10. Model for a DC Machine [4]

With Figure 9 as a reference, the cause and effect equations for the DC Motor are:

$$\frac{di_a(t)}{dt} = \frac{1}{L_a} e_a(t) - \frac{R_a}{L_a} i_a(t) - \frac{1}{L_a} e_b(t)$$

$$T_m(t) = K_i i_a(t)$$

$$e_b(t) = K_b \frac{d\theta_m(t)}{dt} = K_b \omega_m(t)$$

$$\frac{d^2\theta_m(t)}{dt^2} = \frac{1}{J_m} T_m(t) - \frac{1}{J_m} T_L(t) - \frac{B_m}{J_m} \frac{d\theta_m(t)}{dt}$$

Equations 11

According to Equations 11, a Block Diagram for a DC motor should be like this:

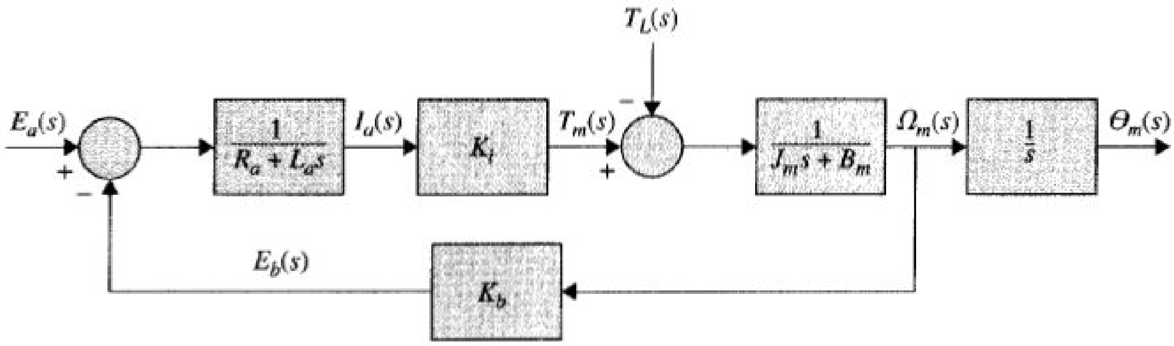

Figure 11. Model for a DC Machine [3]

Basic Types of DC Machines.

1. Separate Winding Excitation (Figure 7)
2. Shunt-Connected dc Machine

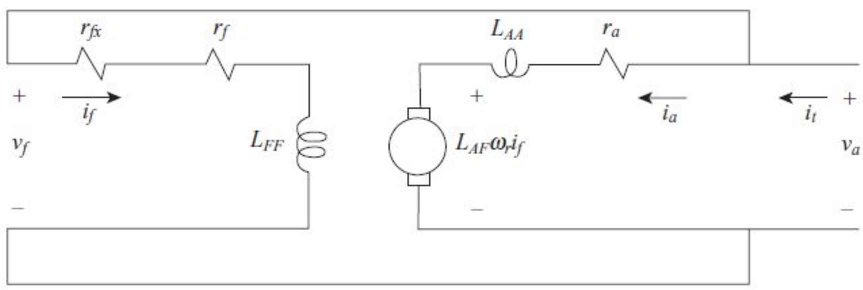

Figure 12. Shunt-Connected DC Machine [1]

3. Series-Connected dc Machine

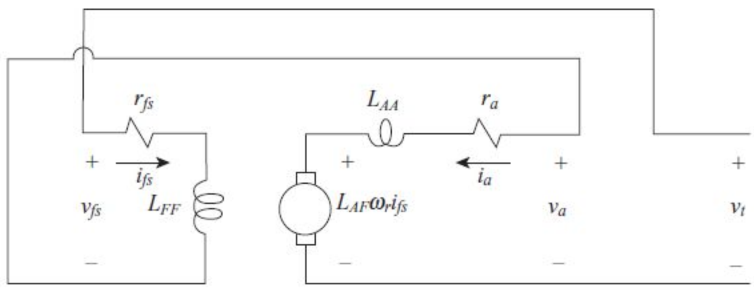

Figure 13. Series-Connected DC Machine [1]

4. Compound-Connected dc Machine

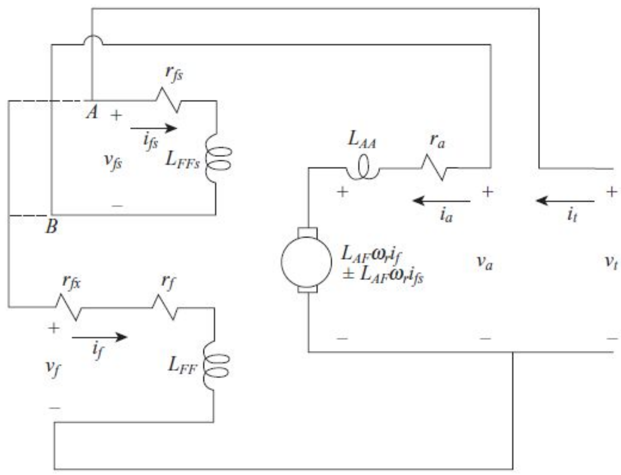

Figure 14. Compound-Connected DC Machine [1]

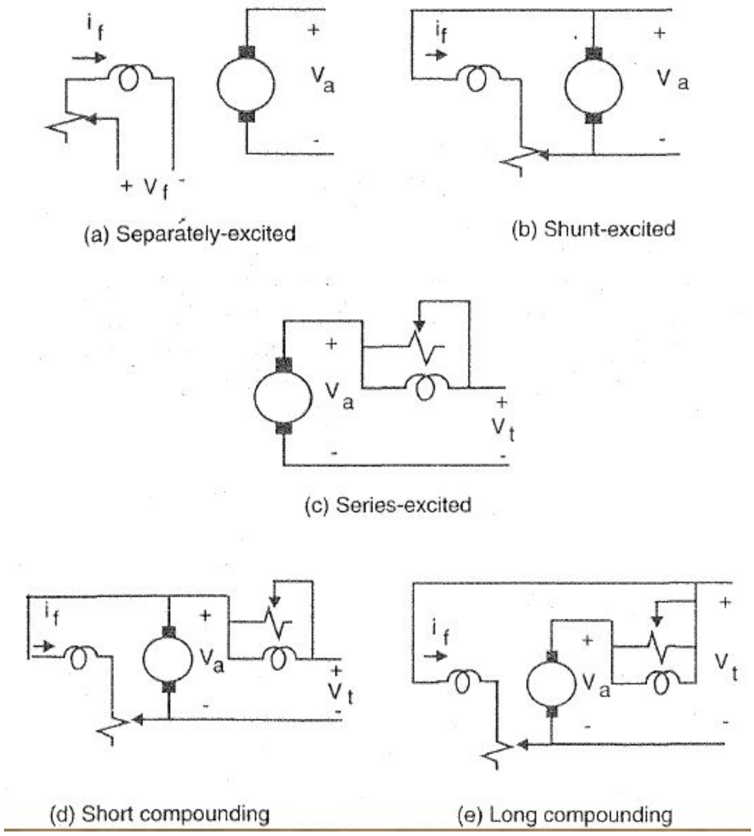

Figure 15. Other notations for DC Machine Types [2]

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